Philosophy 3865 Assignment #1
Due by the start of class on Thursday, March 3rd

You must do problems 1 and 2. Then complete any 3 out of problems 3-7.

1.) Let X' be the complement of X with respect to S. Let Pr be a probability measure on S with $E, F, G \subset S$. Assume that $Pr[E']= 7/13$, $Pr[F]= 6/13$, $Pr[G']= 7/13$, $Pr[E' \cap F]= 3/13$, $Pr[E' \cap G']= 4/13$, $Pr[F \cap G'] = 4/13$, and $Pr[E' \cap F \cap G'] = 2/13$. Find each of $Pr[E \cup F \cup G]$, $Pr[E' \cup F]$ and $Pr[F \cup G]$ and show your work (describe how you got your answer in a way that makes it clear how you could have done a slightly different problem. For example, if you use a venn diagram, tell me the order you filled in the regions and which regions correspond to the answer. If you use the algebraic method, just write the relevant equations, etc.).

2.) You overhear a doctor tell her patient: "Now after the last set of tests, I told you I was 75% sure that you had the antibody in your blood so we decided to do another test. Well, now I can say that I am 95% sure. After all, the test came out positive and the false positive rate on this test is only 10%". Now you know that this doctor is a competent statistician. What can you infer about the false negative rate of the test? If the test had instead came out negative, how should the doctor have revised her beliefs?

3.) Bruno de Finetti believed that the proper axioms for probability theory included only finite additivity and not countable additivity. Explain the difference between the two and carefully describe the 'infinite lottery' example which he took to be an argument to choose between the two. Is this a good argument?

4.) Describe the Principle of Indifference and give an argument against it in terms of the underdescription of the sample space (for example, the cube factory from van Fraassen which is described in Hajek's SEP article). Is this a decisive objection to the principle?

5.) Frequency theories of probability as well as long-run propensity views such as Gillies explain our probability judgments in terms of frequencies within some kind of reference class or long-run sequence. Such views believe that single-case probabilities either don't make sense or don't explain our judgments in real cases. Some other views of probability believe that single-case probabilities do exist and that they ground predictions about long-run frequencies via theorems such as the laws of large numbers. What is the proper relationship between single-case and long-run expected frequencies?

6.) Do 6a through 6d.
6a: Imagine that I post the following betting odds: 4:1 for the proposition that P. 3:1 for the proposition that Q. And 3:1 against P&Q. Describe a set of three bets all at stakes of one dollar and all which I consider to be fair [explain why I consider them fair] and prove that if I take all three bets I am guaranteed to lose money no matter what.

6b: Assume P implies ~Q and that my betting odds for P are a:b and my odds for Q are c:d. What do my betting odds for PvQ have to be to guarantee that I am not subject to a Dutch Book? (You may assume that if my betting ratios are probabilities I am not subject to a Dutch Book).

6c: Assume that my betting odds for PvQ are not as given in the previous problem. Show that for any finite value $M there is a set of three bets that I consider fair such that if I take all three bets I am guaranteed to lose at least $M no matter what.

6d: Repeat question 1c: except assume that I do not accept bets that I consider to be fair, only that I consider to be to my advantage (subjected expected value is greater than $0. Show that I am still subject to a Dutch Book where I will lose at least $M no matter what.

7) In chapter 13, Hacking describes various ways of getting at an agent' s personal probabilities by asking about preferences between various gambles. The Dutch book argument apparently assumes that for any particular proposition X, an agent will have some degree of belief in X which can be appropriately determined by determining which bets that agent considers fair which is then interpreted as 'would be willing to take either side of the relevant bet'. The argument then proceeds to claim that these degrees of belief must be probabilities. Imagine that for some particular agent and for some X, there is no bet that the agent would be willing to take either side of (Kyburg and the example of raining today is one possibility). Does this mean that there is no bet that the agent considers fair? Does it mean that the agent has no degree of belief with respect to X? Does this represent any kind of failure of the Dutch Book argument? Or something else?